IIR Butterworth Low-High Pass Optical Filters Design

Elham Jasim Mohammad

Abstract— In this study, the efforts have been made to introduce the concept of filtering, describes Butterworth and Infinite Impulse Response (IIR) filters, and how it can be designed using MATLAB. The types of IIR filters like Butterworth low-pass and high-pass filters are designed to generate their magnitude response and filter coefficients. The main goal of this work is to obtain an optimized filter response along with the filter coefficients.

Index Terms— Band-pass, Band-stop, Butterworth, High-pass, Low-pass.

1 Introduction

OPTICS filters are devices that selectively transmit light of different wavelengths or have interference coatings. The simplest, physically, is the absorptive filter; interference can be quite complex. Optical filters selectively transmit light in a particular range of wavelengths. They can usually low-pass or high-pass [1], [2].

An filter could correctly be called low-pass, but conventionally is described as long-pass (low frequency is long wavelength), to avoid confusion. A low-pass filter is an electronic filter that passes low frequency signals and attenuates (reduces the amplitude of) signals with frequencies higher than the cutoff frequency. The actual amount of attenuation for each frequency varies from filter to filter. A band-pass filter is a combination of a low-pass and a high-pass. A high-pass filter is an electronic filter that passes high frequency signals but attenuates signals with frequencies lower than the cutoff frequency [1]. The actual amount of attenuation for each frequency also varies from filter to filter. A high-pass filter is usually modeled as a linear time invariant system. It is sometimes called a low-cut filter or bass-cut filter.

The theoretical filter design problem involves the determination of a set of filter coefficients to meet a set of design specifications. These specifications typically consist of the width of the pass-band and the corresponding gain, the width of the stop-band and the attenuation therein; the band edge frequencies and the peak ripple tolerable in the pass-band and stop-band [3].
Measure both the pass-band ripple and the stop-band attenuation in decibels (dB).

In this paper, two types of Infinite Impulse Response (IIR) Butterworth filters designed. IIR filter possess certain properties, which make them the preferred design choices in numerous situations. The ratio of the amplitude of the output wave to the amplitude of the input wave defines what is called the amplitude response of the filter, and can be measured using a sample sine wave.

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2 Infinite Impulse Response Filters

Infinite Impulse Response filters, known as recursive filters operate on current and past input values and current and past output values. Theoretically, the impulse response of an IIR filter never reaches zero and is an infinite response. A recursive filter is one which in addition to input values also uses previous output values [4], [5].
The IIR filter can realize both the poles and zeroes of a system because it has a rational transfer function, described by polynomials in z in both the numerator and the denominator. The equation below defines the direct form transfer function of an IIR filter [5]:

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} / \sum_{k=1}^{N} a_k z^{-k} \quad (1)$$

A filter implemented by directly using the structure defined by the above equation. Where, *M* and *N* are order of the two polynomials, a_k and b_k are the reverse and forward coefficients of the IIR filter. It can be written in the form of general difference equation as follows [5]:

$$y(n) = \sum_{k=0}^{M} b_k x(n-k) + \sum_{k=1}^{N} a_k y(n-k) \quad (2)$$

IIR filters can be expanded as infinite impulse response filters. In designing IIR filters, cutoff frequencies of the filters should be mentioned. The order of the filter can be estimated using butter worth polynomial. That's why the filters are named as Butterworth filters. Filter coefficients can be found and the response can be plotted [6].

3 Butterworth Filters

The Butterworth filter is designed to have as flat a frequency response as possible in the pass-band. It is also referred to as a maximally flat magnitude filter. It was first described in 1930 by the British engineer Stephen Butterworth in his paper entitled "On the Theory of Filter Amplifiers".
Butterworth filters are defined by the property that the amplitude response is maximally flat in the pass-band. Butterworth

filters have the following characteristics:

1- Smooth response at all frequencies.
2- Maximal flatness, with the ideal response of unity in the pass-band and zero in the stop-band.
3- Half-power frequency or 3dB down frequency, that corresponds to the specified cutoff frequencies.

The transfer function for Butterworth filter is given by [5]:

$$B(\omega) = 1/\left[1 + (\omega/\omega_0)^{2n}\right]^{1/2} \quad (3)$$

Where ω is the angular frequency in radians per second and n is the number of poles in the filter equal to the number of reactive elements in a passive filter. If ω = 1, the amplitude response of this type of filter in the pass-band is $1/\sqrt{2} \approx 0.707$, which is half power or 3dB.

Higher order Butterworth filters approach the ideal low-pass filter response. Butterworth filters do not always provide a good approximation of the ideal filter response because of the slow roll-off between the pass-band and the stop-band [7], [8].

4 Results And Discussion

The Butterworth filter is one type of optical digital filter design, which is designed to have a frequency response which is as flat as mathematically possible in the pass-band.

The comparison with the same specification, like cutoff frequency, stop frequency, pass-band attenuation, stop-band attenuation, sample rate and order of the filter, transition band is improved for low-pass and high-pass Butterworth Filter as compared using MATLAB are shown below.

Results are derived for Butterworth filter for magnitude response and phase responses are obtained. On comparing, we see that the results obtained are practically deferent from both types.

The statistical results for mean, median, mode and the standard deviation (STD) are calculated.

Figs. 1 (a) is response of Butterworth low-pass filter. The pass-band ripple for this filter =5. (b) is response of Butterworth high-pass filter with pass-band ripple =5.

In low-pass case, mean= -97.81, median= -112.6, mode= 0, and the STD= 57.55.

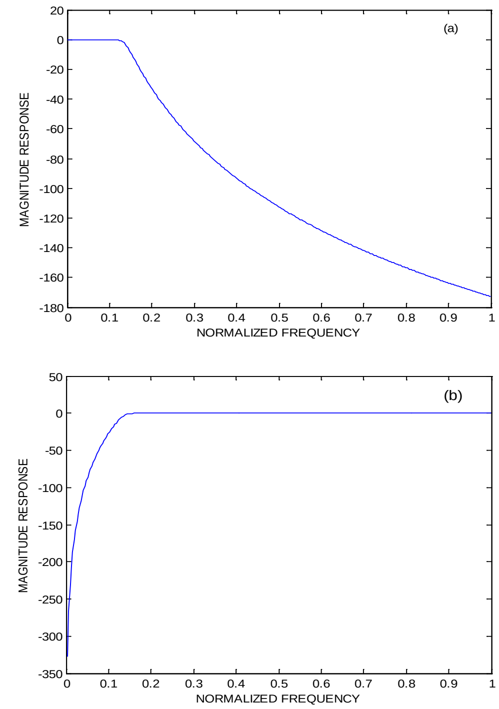
In high-pass case, mean= Inf., median= $-2.403e^{-11}$, mode= $2.005e^{-15}$, and STD= Nan.

Figs. 2 (a) is response of Butterworth low-pass filter. The pass-band ripple for this filter =10. (b) is response of Butterworth high-pass filter with pass-band ripple =10.

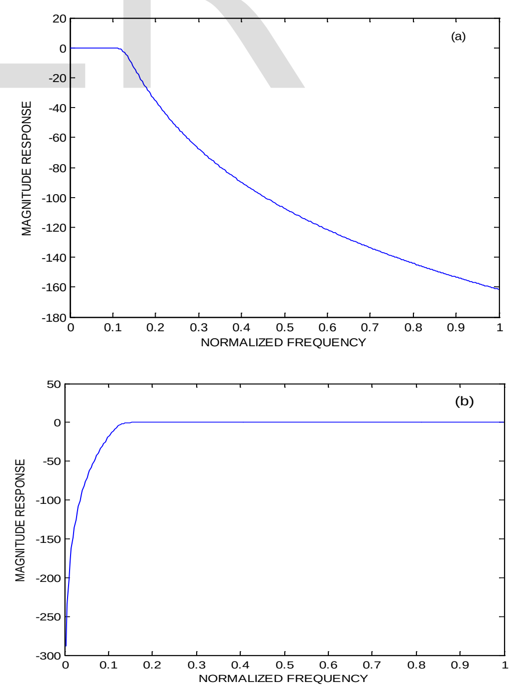
In low-pass case, mean= -93.24, median= -107.3, mode= 0, and STD= 53.16.

In high-pass case, mean= Inf., median= $-8.064e^{-11}$, mode= 0, and STD= Nan.

Filtering involves the manipulation of the spectrum by passing or blocking certain portions of the spectrum, depending on the frequency of those portions. Digital filters are designed using frequency specifications. MATLAB provides different options for digital filter design, which includes function, calls to filter algorithms and a graphical user interface.

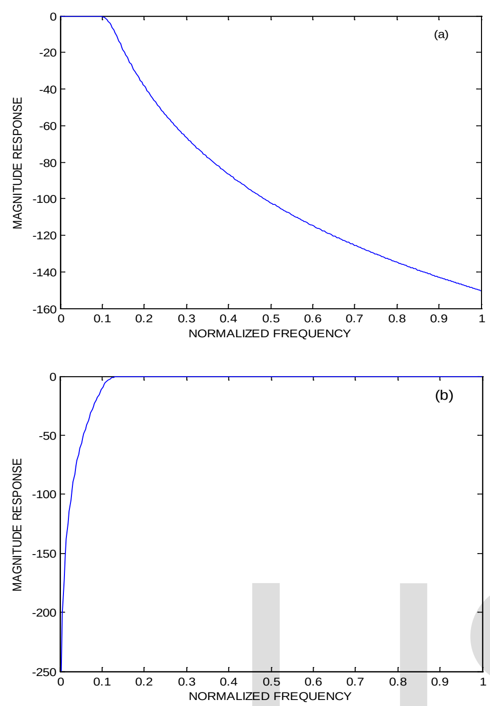


Figs. 1. (a) Magnitude response vs. normalized frequency for low-pass filter (pass-band ripple=5). (b) Magnitude response vs. normalized frequency for high-pass filter (pass-band ripple=5).



Figs. 2. (a) Magnitude response vs. normalized frequency for low-pass filter (pass-band ripple=10). (b) Magnitude response vs. normalized frequency for high-pass filter (pass-band ripple=10).

The frequency response of the Butterworth Filter approximation function is also often referred to as "maximally flat" (no ripples) response because the pass band is designed to have a frequency response which is as flat as mathematically possible.



Figs. 3. (a) Magnitude response vs. normalized frequency for low-pass filter (pass-band ripple=15). (b) Magnitude response vs. normalized frequency for high-pass filter (pass-band ripple=15).

The frequency response of the Butterworth Filter approximation function is also often referred to as "maximally flat" (no ripples) response because the pass band is designed to have a frequency response which is as flat as mathematically possible.

Figs. 3 (a) is response of Butterworth low-pass filter. The pass-band ripple for this filter =15. (b) is response of Butterworth high-pass filter with pass-band ripple =15.

A detailed mathematical analysis of each design was performed. MATLAB code based on the analyses was written.

In low-pass case, mean= -88.74, median= -102.1, mode= 0, and STD= 48.69.

In high-pass case, mean= Inf., median= $-2.705e^{-10}$, mode= $-8.679e^{-15}$, and STD= Nan.

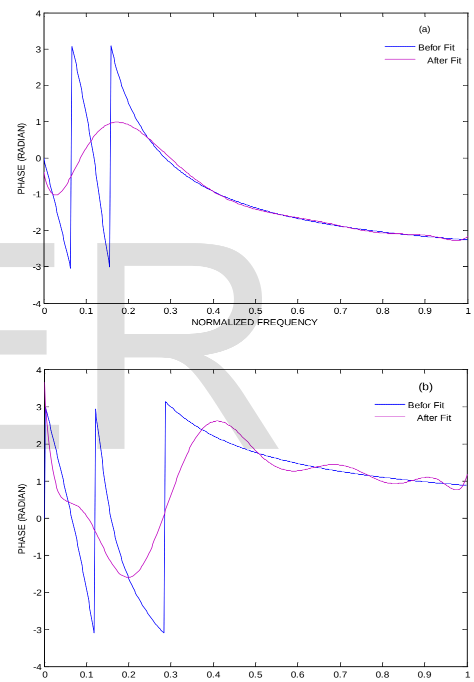
The mathematically simplest and therefore most common approximation is Butterworth filters. Butterworth filters are used mainly because they are easy to synthesize and not because they have particularly good properties. The filter order must be an integer, and we therefore, but not always, select order to the nearest highest integer. The transfer function of Butterworth filters can be calculated by (3). The magnitude function is maximally flat at the origin and monotonically decreasing in both the pass-band and the stop-band as shown above.

However, one main disadvantage of the Butterworth filter is that it achieves this pass-band flatness at the expense of a wide transition band as the filter changes from the pass-band to the stop-band. It also has poor phase characteristics as well. Butterworth filter advantage is that Butterworth filters have a more linear phase response in the pass-band, i.e. the Butterworth filter is able to provide better group delay performance, and also a lower level of overshoot.

Figs. 4 (a) show phase of Butterworth low-pass filter. The pass-band ripple for this filter =5. (b) is phase of Butterworth high-pass filter with pass-band ripple =5.

In low-pass case, mean= -1.062, median= -1.552, mode= -3.057, and STD= 1.323.

In high-pass case, mean= 0.9328, median= 1.209, mode= -3.106, and STD= 1.47.



Figs. 4 (a) Phase vs. normalized frequency for low-pass filter (pass-band ripple=5). (b) Phase vs. normalized frequency for high-pass filter (pass-band ripple=5).

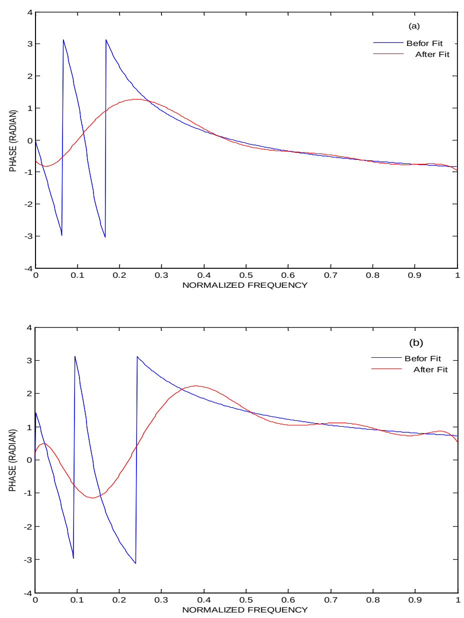
Finally, Figs. 5 (a) show phase of Butterworth low-pass filter. The pass-band ripple for this filter =10. (b) is phase of Butterworth high-pass filter with pass-band ripple =10.

In low-pass case, mean= -0.07398, median= -0.3522, mode= -3.036, and STD= 1.135. In high-pass case, mean= 0.8535, median= 1.041, mode= -3.116, and STD= 1.359.

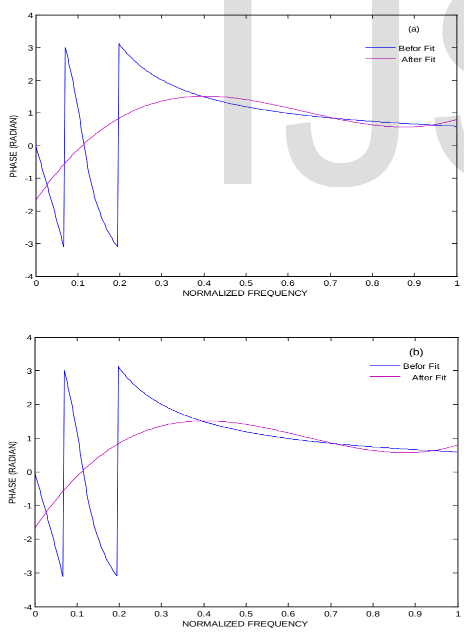
Figs. 6 (a) show phase of Butterworth low-pass filter. The pass-band ripple for this filter =15. (b) is phase of Butterworth high-pass filter with pass-band ripple =15.

In low-pass case, mean= 0.7853, median= 0.8939, mode= -3.116, and STD= 1.277.

In high-pass case, mean= 0.7853, median= 0.8939, mode= -3.116, and STD= 1.277.



Figs. 5 (a) Phase vs. normalized frequency for low-pass filter (pass-band ripple=10). (b) Phase vs. normalized frequency for high-pass filter (pass-band ripple=10).



Figs. 6 (a) Phase vs. normalized frequency for low-pass filter (pass-band ripple=15). (b) Phase vs. normalized frequency for high-pass filter (pass-band ripple=15).

4 Conclusion

Through the results we have obtained in the design of IIR filter, we found that the method used in the design of higher-efficiency through existing evaluation curves above using MATLAB. The flat response within its pass-band and adequate roll-off is two properties for this type of filters. As a result the Butterworth filter may also be known as the maximally flat magnitude filter. The Butterworth filter is often considered as a good all round form of filter which is adequate for many applications, although it does not provide the sharpest cut-off. Butterworth also showed that his basic low-pass filter could be modified to give low-pass, high-pass, band-pass and band-stop functionality. The frequency response of the Butterworth filter is maximally flat (i.e. has no ripples) in the pass-band.

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